BUDDHA SERIES

(Unit Wise Solved Questions & Answers)

Course –B.Sc Math 3rd Year

College – Buddha Degree College

(DDU Code-859)

Department: Science

Subject: Ring theory and Linear Algebra

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Unit - 1

1- The number of binary operations in a ring are

a) 1 b) 2

c) 3 d) 4

2- In which of the following structure may not have unity element:

a) integral domain b) field

c) Ring d) both a and b

3- The characteristic of an integral domain is

a) 0 b) every prime

c) either a or b d) none of these

4- If S1 and S2 are two subrings of a ring R then which of the following set is also a subring of R :

a) S1 U S2 b) S1 - S2

c) S1 ∩ S2 d) S1 + S2

5-If R is a commutative ring then its quotient ring R/S is

a) a ring with unit element b) a commutative ring

c) a commutative ring with unity d) none of these

6-If k(f) is the kernel of homomorphism f from ring R1to the ring R2 then:

a) k(f) is an additive group of R1 b) k(f) ={0}

c) k(f) is a subring of R2 d) none of these

7-Every homomorphic image of a ring (R,+,.) is isomorphic to its:

a). quotient ring b) commutative ring

c) subring d) ideal

8- Every prime field of characteristic zero is isomorphic to the field of :

a). rational numbers b) real numbers

c) Complex numbers d) integers

9- If I is ideal in ring R and a + I = I, then .....

(a) a = 0 (b) a = I (c) a ∈ I (d) none of these

10- The characteristic of the ring (Z4,+,.) of integers modulo 4 is :

a). 1 b) 2

c) 3 d) 4

11- If Fis a field and f(x), g(x) ∈ F [x], then deg. (f(x) g(x)) is equal to:

(a) deg. f(x) (b) deg. g(x)

(c) deg. f(x)-deg. g(x) (d) deg. f(x) + deg. g(x)

12- A polynomial is said to be monic of its leading coefficient is:

a) 2 b) 1

c) 0 d) -1

13. If F is a field, then F[x] is:

(a) a field (b) an integral domain

(c) not a PIR (d) none of these

14. $f(x) = x^2 - 2 \in Z[x]$ is.....

(a) reducible in Z (b) irreducible in Z

(c) irreducible in Q (d) none of these

15. The content of $f(x) = 8x3 + 4x2 + 2x \in Z[x]$ is :
(a) 2 (b) 4 (c) 1 (d) 8

16. The ideal $\langle x \rangle \in Z[x]$ is:
(a) prime ideal (b) maximal ideal
(c) (a) & (b) (d) none of these

17. If R is an integral domain then R [x] is:
(a) an integral domain (b) a field
(c) a unique factorization domain (d) none of these

18- Which of the following is a unique factorization domain?
(a) $Z(\sqrt{-5})$ (b) $Z[\sqrt{-6}]$
(c) $Z[\sqrt{5}]$ (d) $Z[\sqrt{2}]$

19- Every Euclidean Domain is:
(a) Polynomial integral domain (b) unique factorization domain
(c) (a) & (b) (d) none of these

20- If $a + bi$ is not a unit of Z [i], then
(a) $a^2 + b^2 = 1$ b) $a^2 + b^2 > 1$
(c) $a^2 + b^2 = 0$ (d) none of these

21- Which of the following is true for a ring but false for a field?
a) Multiplicative inverse exists for every non-zero element
b) Commutativity of multiplication
c) Existence of additive identity
d) Existence of zero divisors

22- If a ring has characteristic n, then:
a) $n = 1$
b) n must be a prime
c) n is the smallest positive integer such that $n \cdot 1 = 0$
d) n is always zero

23-. Which of the following is preserved under a ring homomorphism?
a) Inverses b) Units c) Addition and multiplication d) Only addition

24- The kernel of a ring homomorphism $f: R \rightarrow S$ is:
a) A subring (b) A maximal ideal
c) A two-sided ideal of R d) A field

25-If I is a maximal ideal in R, then R/I is:
a) A subring b) A field
c) An integral domain d) A ring with zero divisors

Solutions

1. (b) ,**2.** (c) ,**3.** (c) ,**4.** (c) ,**5.** (b) ,**6.** (a) ,**7.** (a) ,**8.** (a) ,**9.** (a) ,**10.** (d) . 11(d) ,**12.** (b) ,1**3.** (b) ,1**4.** (b) ,1**5.** (a) ,1**6.** (a) ,1**7.** (a) ,1**8.** (d) ,1**9.** (c) ,2**0.** (b), 2**1.** (d), 2**2.** (c) , 2**3.** (c) , 2**4.** (c) , 2**5.** (b) .

Unit - II

1- **Eisenstein's criterion applies when:**

a) A prime divides all coefficients except the leading one, and p^2 doesn't divide the constant term

b) All coefficients are divisible by the same prime

c) Constant term is prime

d) Polynomial is monic

2- **A quadratic polynomial over a field is reducible if:**

a) Its discriminant is a perfect square

b) All coefficients are integers

c) It is monic

d) It has no constant term

3-Which of the following tests is commonly used to determine polynomial reducibility?

a) Gauss Lemma

b) Rational Root Theorem

c) Fundamental Theorem of Algebra

d) Fermat's Little Theorem

4- What is the property of a linear functional that states f(ax) = af(x)?

a) Linearity

b) Additivity

c) Homogeneity

d) Continuity

5- What is the dual space of a vector space V?

a) The space of all linear functionals on V

b) The space of all linear transformations from V to another vector space

c) The space of all nonlinear functionals on V

d) The space of all nonlinear transformations from V to another vector space

6- What is the relationship between a bilinear form and a quadratic form?

a) A bilinear form is a quadratic form

b) A quadratic form is a bilinear form

c) A bilinear form can be used to define a quadratic form

d) A quadratic form can be used to define a bilinear form

7- What is a bilinear form?

a) A linear transformation from a vector space to its field of scalars

b) A function that assigns a scalar to each pair of vectors in a vector space

c) A quadratic form on a vector space

d) A linear functional on a vector space

8- What is the definition of a norm?

a) A function that assigns a scalar to each vector in a vector space

b) A function that assigns a scalar to each pair of vectors in a vector space

c) A function that assigns a length to each vector in a vector space

d) A function that assigns a distance to each pair of vectors in a vector space

9. The ideal $\langle x \rangle \in Z[x]$ is:

(a) prime ideal (b) maximal ideal

(c) (a) & (b) (d) none of these

10 Which of the following is a unique factorization domain?

(a) Z ($\sqrt{-5}$) (b) Z [$\sqrt{-6}$]

(c) Z$[\sqrt{5}]$ (d) Z$[\sqrt{2}]$

11. If W1 and W2 are subspaces of a vector space V, then

(a) W1 ∪ W2 are subspace of V (b) W1 ∩ W2 are subspace of V

(c) W1 - W2 subspace of V (d) W1 + W2 are subspace of V

12- If W is a subspace of finite dimensional vector space V (F), then dim (V/W)=

(a) dim V-dim W (b) dim V+dim W

(c) dim V/dim W (d) dim V.dim W

13. A linear transformation T:U→ V is said to be singular if :

(a) T(x)=0 then x=0 (b) T(x)=0 for some non- zero vector x of V

(c) T is a isomorphism (d) dim U=dimV

14. If a linear transformation T on V transforms a bisis of V to a basis of V then T is :

a) singular (b) non- singular

(c) symmetric (d) non- symmetric

15. Two n∗ n matrices A and b are similar iff there exists a non- singular matrix P such that

(a) $A = PBP^{-1}$ (b) $A = PBP$

(c) $A = PB$ (d) $A = BP$

16-. In an integral domain, if ab=, then:

(a) a=or b=0

(b) a= b

(c) a and b is invertible

(d) a and b are zero divisors

17. A ring is an integral domain if:

(a) It has unity and is finite

(b) It has no zero divisors

(c) Every non-zero element is a unit

(d) It is a field

18. An element p in an integral domain is called irreducible if:

(a) It divides every other element

(b) It can be factored into units only

(c) p= ab implies a or b is a unit

(d) p is a unit

19. A prime element in an integral domain satisfies:

(a) If p|ab, then p|a or p|a or p|b

(b) It has no factors

(c) It is irreducible

(d) It divides only units

20. Every prime element in an integral domain is:

(a) Irreducible

(b) Unit

(c) Zero divisor

(d) Invertible

21- If U and V be two finite dimensional vector space over the same F and let

T:U→ V be a linear transformation. The value of ρ(T)+v(T) is

a) dimU b) dim (U+V) c) dim V d) none of these

22- If V is a finite dimensional vector space over the field F and it is the direct sum of its subspaces W_1 and W_2, what is the value of dim V ?

a) dim W_1 + dim W_2 b) dim W_1 c) dim W_2 d) none of these

23- Let T: $R^2 \rightarrow R^2$ be a linear transformation. If T(1, 0) = (2, 3) and T(0, 1) = (4, 5), then what is T(3, 2)?

a) (10, 15) b) (8, 11) c) (14, 19) d) (6, 9)

24- Let T: $R^3 \rightarrow R^2$ be a linear transformation. Which of the following statements is true?

a) T is one-to-one

b) T is onto

c) T is both one-to-one and onto

d) T is neither one-to-one nor onto

25- What is a linear functional?

a) A linear transformation from a vector space to its field of scalars

b) A linear transformation from a vector space to another vector space

c) A nonlinear transformation from a vector space to its field of scalars

d) A nonlinear transformation from a vector space to another vector space

Solutions

1.(a) ,**2.** (a) ,**3.** (b) ,**4.** **(c), 5. (a) ,6. (c) 7. (b) 8. (c),9.** (a) , **10.**(d) ,11(b) , **12.** (a) ,1**3.** (b) ,1**4.** (b) ,1**5.** (a) , 1**6.** (a) ,1**7.** (b) ,1**8.** (c) ,1**9.** (a) ,2**0.** (a), **21. (a), 22. (a) ,23. (a) 24. (b) 25. (a)**